NASA TECHNICAL NOTE

NASA TN D-6804

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NASA TN D-6804

THE EFFECT OF WALL FRICTION ON MAGNETOHYDRODYNAMIC GENERATOR PERFORMANCE

by Allan R. Bishop

Lewis Research Center
Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MAY 1972

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-6804		
4. Title and Subtitle THE EFFECT OF WALL FRICTION ON MAGNETOHYDRO-DYNAMIC GENERATOR PERFORMANCE		5. Report Date May 1972 6. Performing Organization Code
7. Author(s) Allan R. Bishop		8. Performing Organization Report No. E-6436
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		10. Work Unit No. 112-02 11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note 14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

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17. Key Words (Suggested by Author(s))	18. Distribution Statement
Magnetohydrodynamic generator Wall friction Optimum geometry	Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	20	$3.00

* For sale by the National Technical Information Service, Springfield, Virginia 22151

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SUMMARY

The effect of wall friction on magnetohydrodynamic generator performance is determined by introduction of a wall friction factor into the one-dimensional generator equations. This addition should be useful in improving generator analysis and determining optimum generator geometry. The curves presented in this report can be used to determine the effects of changes in wall friction and generator geometry on generator performance. Wall friction has an increasing effect as the Mach number increases and a decreasing effect as the pressure drop across the generator increases.

INTRODUCTION

Magnetohydrodynamic (MHD) power-generator concepts can be divided into two principal classes: (1) large, high-power, ground-based generators, and (2) lower-power generators for space power applications. In ground-based generator concepts, the channel volumes are large, and magnetic field strengths are high. Wall friction effects on generator efficiency are therefore small, and the correlation between theory and experiment is generally good (refs. 1 and 2). The weight of the MHD generator and its magnet are not serious problems in ground-based generators.

For space power applications, the weight of generator and magnet can be important. Hence, a compact, lightweight system is needed, which implies small channel cross sections where friction effects can be important. Until now, most MHD generator analyses (e.g., ref. 3) for space power applications have neglected the effects of wall friction.

Nonuniformities in the fluid flow field and in electrical current distribution can also affect MHD generator performance. The magnitude of these effects may be smaller or greater than the effects of wall friction, depending on the generator configuration. No attempt has been made to include nonuniformities in this analysis. The generator design, basic assumption, and solution techniques used in this report are similar to those of reference 3, except that a wall friction factor has been introduced into the one-dimensional momentum equation.

ANALYSIS

The generator is assumed to be of the Faraday segmented type, with infinitely fine segments so that no current flows in the Hall direction. The working fluid is assumed to be a perfect gas, with zero viscosity and thermal conductivity. A friction factor is introduced to correct for the drag of the generator walls (ref. 4).

The following procedure is used in this analysis:

(1) The one-dimensional flow equations are written in nondimensional form.

(2) The assumptions of uniform power generation and uniform Joule dissipation along the generator are used to reduce the number of unknown variables.

(3) The variation of the cross-sectional area is determined by requiring minimum generator volume for a given total pressure ratio. A variational method with Lagrange multipliers is used to minimize the volume.

(4) With the area variation known, the generator performance can be found as a function of inlet and load conditions.

All symbols are defined in the appendix, and all quantities are expressed in the International System of Units.

The five one-dimensional equations for the flow are

Continuity:

$$\frac{d}{dx}(\rho u a) = 0 \tag{1}$$

Momentum:

$$\rho u \frac{du}{dx} + \frac{dP}{dx} + jB + \frac{f_F \rho u^2}{2r_h} = 0 \tag{2}$$

Energy:

$$\rho u \frac{dh}{dx} + \rho u^2 \frac{du}{dx} - jE = 0 \tag{3}$$

where

$$h = \frac{\gamma}{\gamma - 1} \frac{P}{\rho} \tag{4}$$

Ohm's law:

$$j = \sigma(uB + E) \tag{5}$$

The last term in equation (2) is the correction for wall friction losses, and is the only addition to the flow equations in reference 3.

These equations can be written in nondimensional form by using the following definitions:

$$A = \frac{a}{a_1} \qquad U = \frac{u}{u_1}$$

$$\Sigma = \frac{\sigma}{\sigma_1} \qquad C = \frac{B}{B_1}$$

$$R = \frac{\rho}{\rho_1} \qquad p = \frac{P}{\rho_1 u_1^2}$$

$$X = \frac{\sigma_1 B_1^2 x}{\rho_1 u_1} \qquad J = \frac{j}{\sigma_1 u_1 B_1}$$

where the subscript 1 indicates channel entrance conditions. Equations (1) to (5) reduce to

$$\frac{dU}{dX} + A \frac{dP}{dX} + JAC + \frac{f_F \rho_1 u_1}{2r_h \sigma_1 B_1^2} U = 0 \tag{6}$$

$$\frac{\gamma}{\gamma - 1} \frac{d}{dX} (AUP) + U \frac{dU}{dX} + JAC \left(U - \frac{J}{\Sigma C}\right) = 0 \tag{7}$$

Define a new parameter F_o such that

$$F_o = \frac{f_F \rho_1 u_1}{2r_h \sigma_1 B_1^2}$$

Note that r_h is the hydraulic radius, which is one-fourth of the hydraulic diameter and is assumed to be constant. Equation (6) then becomes

$$\frac{dU}{dX} + A\frac{dP}{dX} + JAC + F_o U = 0 \tag{8}$$

It is now assumed that

$$JC = J_{ent}$$

and

$$\frac{J}{\Sigma C} = J_{ent}$$

where J_{ent} is a constant. The use of these assumptions results in uniform power generation and uniform Joule dissipation along the generator.

The constant J_{ent} is related to the load parameter K by the equation

$$J_{ent} = 1.0 - K$$

where K is the ratio of actual output voltage to open circuit voltage. The value of J_{ent} is 0 at open circuit and 1 at short circuit.

With these two additional assumptions, equations (7) and (8) reduce to

$$\frac{\gamma}{\gamma - 1}\frac{d}{dX}(AUP) + U\frac{dU}{dX} + J_{ent}A(U - J_{ent}) = 0 \tag{9a}$$

and

$$\frac{dU}{dX} + A\frac{dP}{dX} + J_{ent}A + F_o U = 0 \tag{9b}$$

The total pressure in dimensionless variables is

$$P_T = P\left(1 + \frac{\mu}{2}\frac{U}{AP}\right)^{1/\mu} \tag{10}$$

where

$$\mu = \frac{\gamma - 1}{\gamma}$$

Let

$$q_T = (P_T)^{\mu}$$

and

$$q = (P)^{\mu}$$

Then, solving for U and substituting into equation (9b) yields

$$dU + A\,dP + \left[J_{ent} + \frac{2}{\mu} F_o(q_T - q)q^{1/\mu - 1}\right] A\,dX = 0 \tag{11}$$

It is still necessary to specify the variation in cross-sectional area as a function of the distance along the generator length. It is important to maintain a high generator efficiency for a specified total pressure ratio. The only energy loss considered in this analysis is Joule heating, which varies with generator volume and is assumed uniform along the duct. It seems reasonable, therefore, to minimize the generator volume for a specified total pressure ratio, since this will minimize the energy loss. The procedure has also been followed in reference 3.

The volume τ of the generator is given by the integral of the area

$$\tau = \int_0^l a\,dx$$

Define a nondimensional volume V as

$$V = \frac{\sigma_1 B_1^2 \tau}{a_1 \rho_1 u_1}$$

$$= \int_0^L A\,dX \tag{12}$$

Substituting equation (11) into (12) yields

$$V = - \int \frac{dU + A\, dP}{J_{ent} + \frac{2}{\mu} F_o(q_T - q)q^{1/\mu - 1}} \tag{13}$$

Define

$$F = \frac{\frac{dU}{dq_T} + A \frac{dP}{dq_T}}{J_{ent} + \frac{2}{\mu} F_o(q_T - q)q^{1/\mu - 1}}$$

then V is

$$V = - \int_{q_{T_1}}^{q_{T_{exit}}} F\, dq_T \tag{14}$$

This is the integral to be minimized, subject to the constraints of equations (9) and (11). Eliminating $A\, dX$ from equations (9) and (11) yields

$$\left(dU + \frac{U}{2} \frac{dq}{q_T - q}\right) J_{ent}(U - J_{ent}) = \left[J_{ent} + \frac{2F_o}{\mu}(q_T - q)q^{1/\mu - 1}\right]$$

$$* \left[\frac{U}{2(q_T - q)}\left(2q\, dU + \frac{Uq_T\, dq}{q_T - q} - \frac{Uq\, dq_T}{q_T - q}\right) + U\, dU\right] \tag{15}$$

At the minimum value of V, the variation in V must vanish (ref. 5, p. 74).

$$\delta V = - \frac{1}{2} \frac{\partial F}{\partial U'}\bigg|_L \delta U - \frac{1}{2} \frac{\partial F}{\partial q'}\bigg|_L \delta q - \frac{1}{2} F\bigg|_L \delta q_T$$

$$+ \int_{q_{T_1}}^{q_{T_{exit}}} \left[\left(\frac{\partial F}{\partial U} - \frac{d}{dq_T} \frac{\partial F}{\partial U'}\right)\delta U + \left(\frac{\partial F}{\partial q} - \frac{d}{dq_T} \frac{\partial F}{\delta q'}\right)\delta q\right] dq_T \tag{16}$$

where

$$U' = \frac{dU}{dq_T}$$

$$q' = \frac{dq}{dq_T}$$

The third term on the right side of equation (16) is zero, since the exit total pressure q_T is fixed. The first two terms may be reduced to the constraint equation (15) with dq_T set equal to zero, and their sum must be zero. Therefore, the integral term must also vanish. The minimization of the volume may then be handled as a fixed end-point problem with a constraint. With the introduction of the new variable

$$v = \frac{U}{q_T - q} \tag{17}$$

and its derivative

$$v' = \frac{dv}{dq_T} = \frac{U'}{q_T - q} - \frac{v}{q_T - q}(1 - q')$$

the integrand of equation (14) becomes

$$F = \frac{v'(q_T - q) - q'\frac{v}{2} + v}{J_{ent} + \frac{2F_o}{\mu}(q_T - q)q^{1/\mu - 1}} \tag{18}$$

and the constraint equation is

$$\varphi = \left[v'(q_T - q) - \frac{v}{2}q'\right]\left[J_{ent}(J_{ent} + qv) + \frac{2}{\mu}F_o v q_T (q_T - q)q^{1/\mu - 1}\right]$$

$$+ v\left[J_{ent}\left(J_{ent} + \frac{vq}{2}\right) + \frac{F_o v}{\mu}(q_T - q)(2q_T - q)q^{1/\mu - 1}\right] = 0 \tag{19}$$

Applying the technique of Lagrange multipliers (ref. 5, p. 129), introduce $H = F + \lambda\varphi$, where λ is the undetermined Lagrange multiplier. The integral I will be minimized if the equations

$$\left.\begin{aligned}
&\frac{\partial H}{\partial q} - \frac{d}{dq_T}\frac{\partial H}{\partial q'} = 0 \\
&\frac{\partial H}{\partial v} - \frac{d}{dq_T}\frac{\partial H}{\partial v'} = 0 \\
&\varphi = 0
\end{aligned}\right\} \tag{20}$$

are satisfied. Substituting in H and taking the appropriate derivatives gives three equations for q, v, and λ:

$$\begin{pmatrix} \alpha & 0 & \beta \\ 0 & -\alpha & \gamma \\ \gamma & \beta & 0 \end{pmatrix}\begin{pmatrix} v' \\ q' \\ \lambda' \end{pmatrix} + \begin{pmatrix} A^* \\ B^* \\ C^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{21}$$

where

$$\alpha = -\frac{\dfrac{J_{ent}}{2} + \dfrac{2F_o(q_T - q)}{\mu^2}\left[q_T(1 - \mu) - q\left(1 - \dfrac{\mu}{2}\right)\right]q^{1/\mu-2}}{\left[J_{ent} + \dfrac{2F_o}{\mu}(q_T - q)q^{1/\mu-1}\right]^2}$$

$$+ \lambda J_{ent}\left[v(q_T - q) - \frac{J_{ent}}{2}\right] + \frac{2\lambda F_o v q_T}{\mu^2}(q_T - q)\left[q_T(1 - \mu) - q\right]q^{1/\mu-2}$$

$$\beta = \frac{v}{2}\left[J_{ent}(J_{ent} + qv) + \frac{2F_o v q_T}{\mu}(q_T - q)q^{1/\mu-1}\right]$$

$$\gamma = -(q_T - q)\left[J_{ent}(J_{ent} + qv) + \frac{2F_o v q_T}{\mu}(q_T - q)q^{1/\mu-1}\right]$$

$$A^* = -\frac{vF_o\left[2q_T(1-\mu) - q(2-\mu)\right]q^{1/\mu-2}}{\mu^2\left[J_{ent} + \frac{2F_o}{\mu}(q_T - q)q^{1/\mu-1}\right]^2}$$

$$+ \lambda v^2\left\{\frac{J_{ent}}{2} + \frac{F_o}{\mu^2}\left[2q_T^2(1-\mu) - qq_T(3-2\mu) + q^2\right]q^{1/\mu-2}\right\}$$

$$B^* = \frac{2F_o(q_T - q)q^{1/\mu-1}}{\mu\left[J_{ent} + \frac{2F_o}{\mu}(q_T - q)q^{1/\mu-1}\right]^2} - \frac{2\lambda F_o v q_T}{\mu}(q_T - q)q^{1/\mu-1}$$

$$C^* = -v\left[J_{ent}\left(\frac{vq}{2} + J_{ent}\right) + \frac{F_o v}{\mu}(q_T - q)(2q_T - q)q^{1/\mu-1}\right]$$

The matrix equation (21) is singular and unless

$$G = \alpha C^* + \beta B^* - \gamma A^* = 0 \tag{22}$$

there are no solutions. Equation (22), which is required for the matrix to have a solution defines the Lagrange multiplier λ.

With the conditions $G = 0$ and $dG = 0$, the matrix can be rewritten as

$$\begin{pmatrix} \alpha & 0 & \beta \\ 0 & -\alpha & \gamma \\ \frac{\partial G}{\partial v} & \frac{\partial G}{\partial q} & \frac{\partial G}{\partial \lambda} \end{pmatrix}\begin{pmatrix} v' \\ q' \\ \lambda' \end{pmatrix} + \begin{pmatrix} A^* \\ B^* \\ \frac{\partial G}{\partial q_T} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{23}$$

The solution to this system of equations determines the variation of v, q, and λ which minimizes the volume. Substitution into equation (10) gives the proper area variation, and the volume can be determined from equation (12).

RESULTS AND DISCUSSION

Matrix inversion by the Gauss-Jordon reduction method was used to solve equation (23). Typical values for the dimensionless volume and area ratio are shown as functions of total pressure ratio in figure 1, for an entrance Mach number of 1.0 and J_{ent} of 0.25. The solid lines are the solution for zero friction, $F_o = 0.0$, while the broken lines are the solutions for $F_o = 0.1$.

The curves for $F_o = 0.0$ are identical to the curves in figure 7 of reference 3. The value $F_o = 0.1$ was determined from the following typical entrance conditions: $u_1 = 800$ m/sec, $\rho_1 = 0.25\ \mathrm{kg/m^3}$, $f_F = 2.5\times10^{-3}$, $r_h = 2.5$ cm, $B_1 = 1.0$ tesla, $\sigma_1 =$ ohm/m. Since the area and volume are variables, a change in the friction can alter the area and volume for a given pressure drop. Note that the area increases slightly but the volume decreases as F_o is increased at constant pressure.

In figure 2, the nondimensional generator length X_L is plotted as a function of the total pressure ratio for two values of F_o. As might be expected, the same pressure

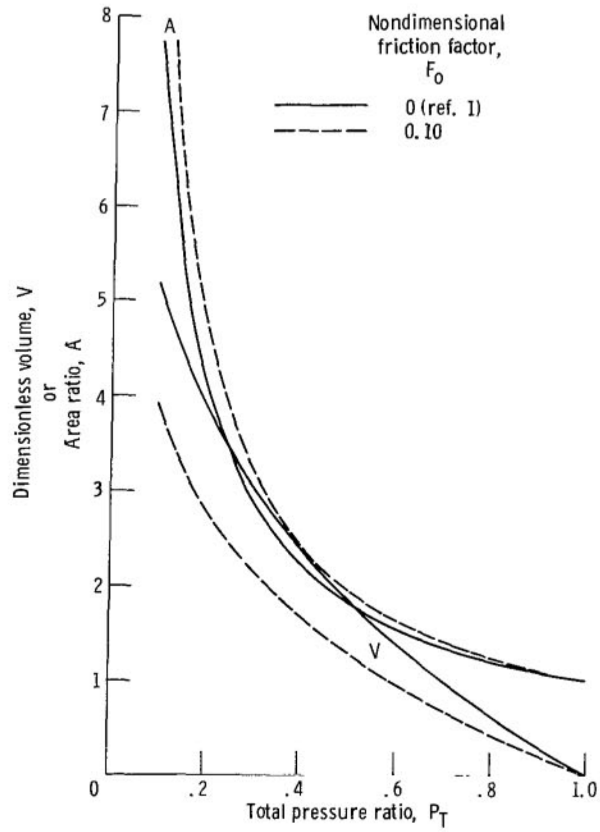


Figure 1. - Dimensionless volume and area ratio as functions of total pressure ratio. Entrance Mach number, 1.0; dimensionless load current, 0.25.

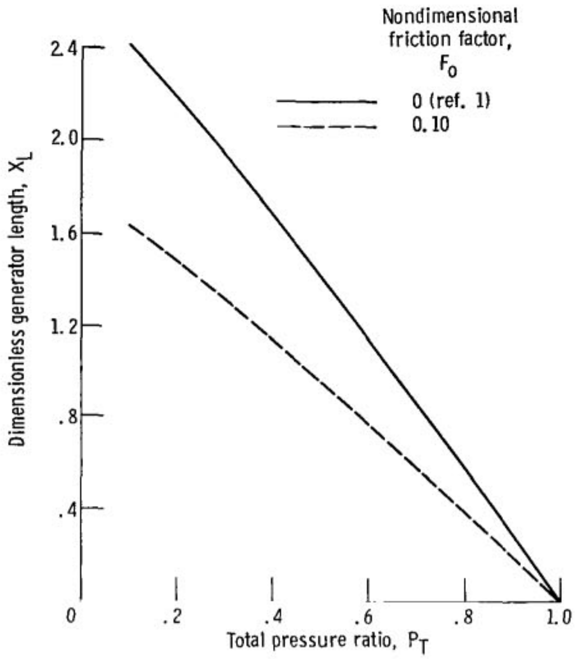


Figure 2. - Dimensionless generator length as a function of total pressure ratio. Entrance Mach number, 1.0; dimensionless load current, 0.25.

drop occurs in a shorter length as the friction forces are increased (F_0 increased). The increase in cross-sectional area (fig. 1) is not enough to compensate for the increased friction. As a result, the volume is smaller at the increased friction.

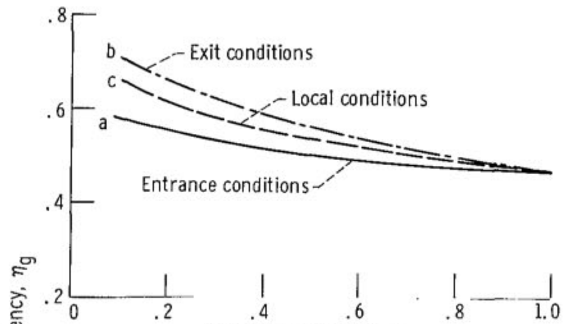
In most MHD generators, an important design factor is the generator efficiency. Here the efficiency means the isentropic efficiency, the calculated change in enthalpy divided by the isentropic change in enthalpy between the same total pressure limits. Sutton and Sherman (ref. 6) discuss the effects of wall friction on generator efficiency in terms of a polytropic efficiency η_p. The appropriate expression for η_p given by equation (14.98) in reference 6 is

$$\eta_p = \frac{1}{\left[\dfrac{\dfrac{J_{ent}}{U} + \dfrac{F_o}{A}}{\left(\dfrac{J_{ent}}{U} - 1\right)\left(\dfrac{J_{ent}}{U}\right)} - 1\right]\left(1 + \dfrac{\gamma - 1}{2} M^2\right) + 1}$$

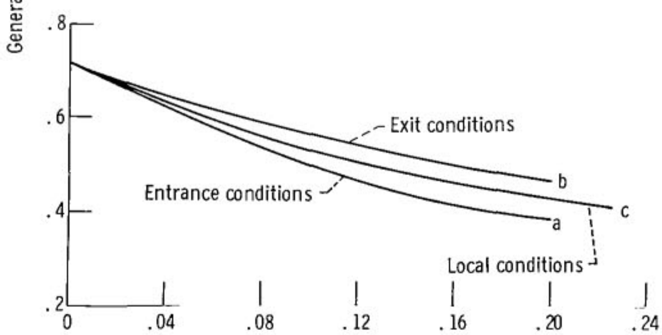
where the notation of reference 6 has been changed to correspond to the notation used in this report. Since U, A, and M may vary as the pressure ratio changes along the length of the generator, η_p is actually a local polytropic efficiency. The generator efficiency η_g, from reference 6, is

$$\eta_g = \frac{1 - P_T^{\eta_p(\gamma-1)/\gamma}}{1 - P_T^{(\gamma-1)/\gamma}}$$

This efficiency depends on the total pressure ratio, both explicitly and implicitly, since η_p also can depend on pressure ratio. In order to calculate the generator efficiency as a function of pressure ratio, it is necessary to include the polytropic efficiency variation. The effect of the variation in polytropic efficiency can be seen in figure 3(a). The generator efficiency is calculated for three cases: (a) a constant polytropic efficiency evaluated at the entrance condition, (b) a constant polytropic efficiency evaluated at the exit condition, and (c) the correct local polytropic efficiency evaluated at each pressure from the entrance to the exit. Curve c fits between curves a and b, just as one would expect.



(a) Friction factor, 0.1.



(b) Total pressure ratio, 0.5.

Figure 3. - Generator efficiency as a function of total pressure ratio and friction factor. Entrance Mach number, 1.0; dimentionless load current, 0.25.

The overall generator efficiency is therefore dependent on the pressure ratio, which in turn is dependent on generator length. The efficiency is also a function of hydraulic radius. The efficiency is plotted in figure 3(b) as a function of F_o (which is inversely proportional to the hydraulic radius). The curves labeled a, b, and c correspond to those in figure 3(a). Again, with the use of the polytropic efficiency at each point, curve c fits between curves a and b, which are based on entrance and exit conditions.

Thus, the efficiency is affected by both hydraulic radius and length, but a new nondimensional number that combines both factors can be defined. Multiplying F_o by the nondimensional generator length yields

$$F_o \cdot X_L = \frac{f_F \rho_1 u_1}{2 r_h \sigma_1 B_1^2} \cdot \frac{\sigma_1 B_1^2 L}{\rho_1 u_1}$$

$$= \frac{f_F L}{2 r_h}$$

where L is the generator length, f_F is the friction coefficient, and r_h is the hydraulic radius. This parameter is plotted in figure 4 as a function of J_{ent} for constant generator efficiency. Curves are plotted for two pressure ratios, 0.5 and 0.1. At pressure ratios lower than 0.1, the area ratio increases so rapidly (fig. 1) that a one-dimensional analysis no longer applies. In figure 4(a) the entrance Mach number is 0.5, in figure 4(b) it is 1.0, and in figure 4(c) it is 2.0.

From the curves in figure 4, we can determine the effect of friction on the generator efficiency. Clearly, as the friction coefficient increases (or the generator length L increases, or the generator hydraulic radius r_h decreases), the generator efficiency goes down at constant J_{ent}. To maintain constant efficiency, J_{ent} could be lowered, but with the penalty of decreased output power or increased generator size.

As an example, the 10-MW cesium-seeded generator mentioned in reference 7 has an inlet gas temperature of 2500 K at 10 atmospheres and is 1/2 meter long. Assuming a square cross section, an aspect ratio (length divided by square root of entrance area) of 3, an 80-percent efficiency, and operation near Mach 1, the required entrance magnetic field for a frictionless generator is 8.4 tesla. If the friction coefficient f_F is increased to 0.003, the magnetic field required to maintain constant efficiency increases to 10.0 tesla. If the magnetic field is not increased, the generator efficiency will be reduced to approximately 70 percent. A friction coefficient of 0.003 is within the current state of the art, but some care is required to achieve it.

Generator efficiency is much more sensitive to wall friction as the Mach number increases, as can readily be seen by comparing figure 4(a) with figures 4(b) and (c). To

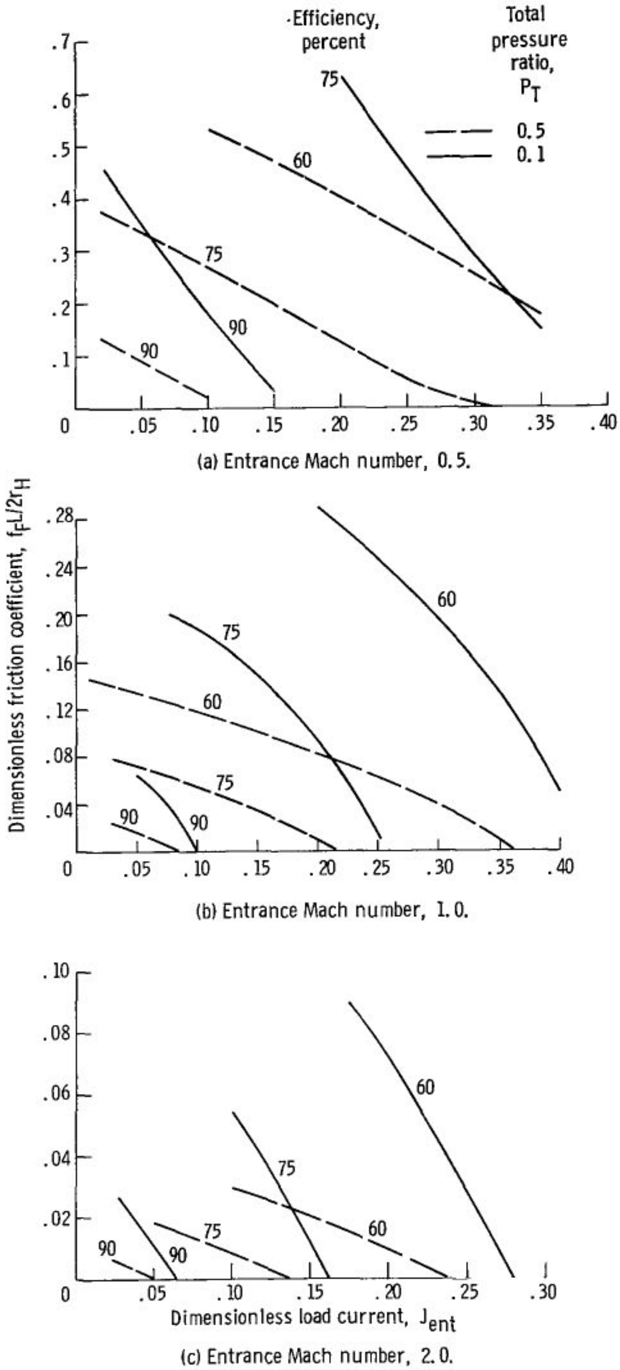


Figure 4. - Dimensionless friction coefficient as a function of dimensionless load current, with efficiency as a parameter.

maintain constant efficiency, a larger change in J_{ent} is required at Mach 2.0 than at Mach 0.5 (fig. 5), for the same change in friction coefficient. The results of this analysis indicate that smooth walls and proper geometry are much more important in a supersonic generator than in a subsonic one.

The expansions to a pressure ratio of 0.1 seem to be less sensitive to friction than the expansion to a pressure ratio of 0.5. Note that in figure 4 the 0.1 expansion has a steeper slope than the expansion to 0.5. The increased area at lower pressures (fig. 1) and the resulting reduction in wall friction effects may explain this difference.

None of the curves in figures 4 or 5 have been extended to $J_{ent} = 0$, because a solution could not be obtained in this region. The mathematical difficulty was a zero value for the determinant in the matrix inversion procedure. Whether some physical phenomenon was occurring or whether a mathematical instability of the analytical procedure was at fault could not be determined. For some, but not all, cases the problem was asso-

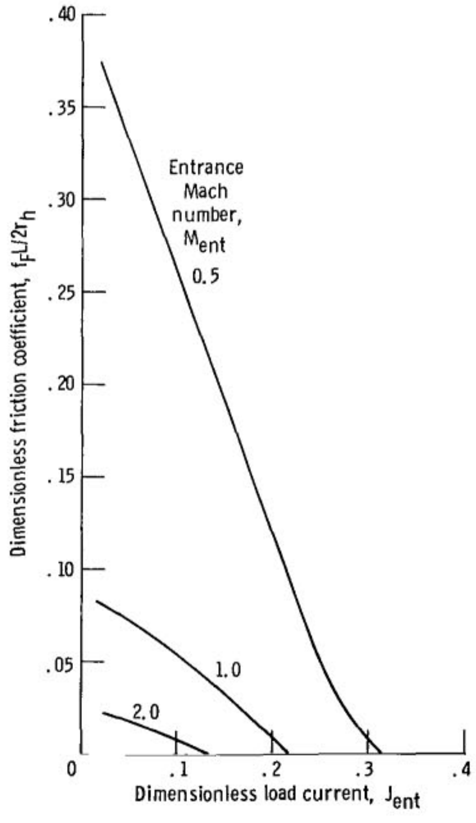


Figure 5. - Dimensionless friction coefficient as a function of entrance current density ratio, with Mach number as a parameter. Efficiency, 0.75; total pressure ratio, 0.5.

ciated with an adverse pressure gradient in the flow. A similar situation was noted in the analysis of reference 1, but since the analytical methods are quite different, a direct comparison was not possible.

CONCLUSIONS

The addition of wall friction to the MHD generator equations should be useful in improving generator analysis. The curves presented in this report can be used to determine optimum generator geometry and the effect of changes in wall friction and generator geometry on generator performance.

The wall friction seems to have a greater effect as the Mach number increases, and a smaller effect at large pressure drops across the generator. The latter effect may be the result of the area variation used in this analysis.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, March 30, 1972,
112-02.

APPENDIX - SYMBOLS

A	generator area ratio, a/a_1
A^*, B^*, C^*	terms defined in eq. (21)
a	generator cross-sectional area
B	magnetic field
C	magnetic field ratio, B/B_1
E	generator electric field
F	integrand in minimum volume integral
F_o	nondimensional friction factor
f_F	wall friction coefficient
G	parameter defined by eq. (22)
H	function defined by eq. (20)
h	enthalpy
J	dimensionless current density
J_{ent}	dimensionless load current
j	current density
K	load parameter
L	generator length
P	dimensionless pressure
p	pressure
q	variables defined in eq. (10)
R	density ratio ρ/ρ_1
r_h	hydraulic radius; ratio of cross-sectional area to perimeter
U	velocity ratio u/u_1
u	velocity
V	dimensionless generator volume
v	variable defined in eq. (17)
X	dimensionless length
X_L	dimensionless generator length

α, β, γ	terms defined in eq. (21)
γ	ratio of specific heats
δ	variational symbol
η_g	generator efficiency
η_p	polytropic efficient
λ	Lagrange multiplier
μ	function of specific heat ratio defined in eq. (10)
ρ	working fluid density
Σ	electrical conductivity ratio σ/σ_1
σ	electrical conductivity
τ	generator volume
φ	constraint equation defined in eq. (19)

Subscripts:

exit	exit conditions
L	generator exit value
T	stagnation property
1	entrance conditions

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